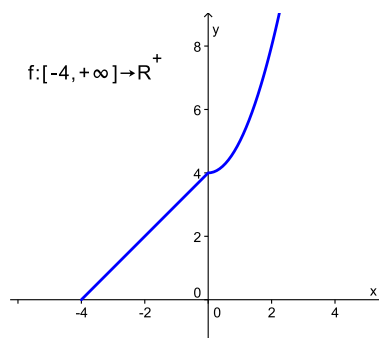
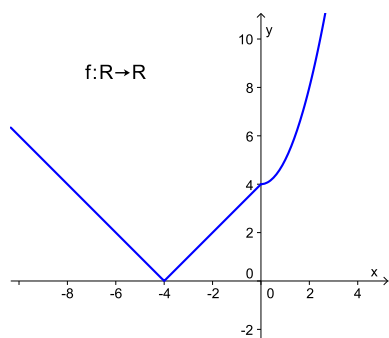


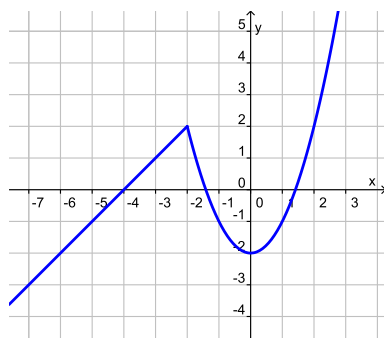
Soluzioni

Capitolo 1

1.25 Nella prima definizione f non è biunivoca, nella seconda lo è.



1.26 $f^{-1}(0) = \{-4, -\sqrt{2}, \sqrt{2}\}$, $f^{-1}(-3) = \{-7\}$. Non è biunivoca.



1.27 (a) Funzione pari, limitata. (b) Funzione dispari, iniettiva, suriettiva. (c) Funzione pari sempre positiva.

- 1.28
- $|\ln^2 7|$
 - $|x| - 7$
 - $\ln^2(x^2 - x) - 1$
 - $\ln^4 x - \ln^2 x$
 - $7 + |x|$
 - $|x^2 - x| - (x^2 - |x|)$
 - $|\sqrt{x}| \ln^2(x^2 - x)$
 - $5 + x^2$
 - $|\ln x| + |\ln^2 x|$

1.29 $a = 2, b = -1$

1.30 -1

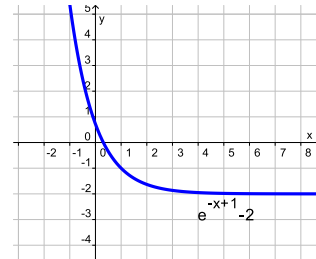
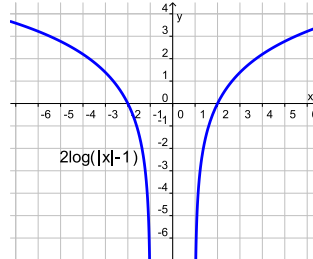
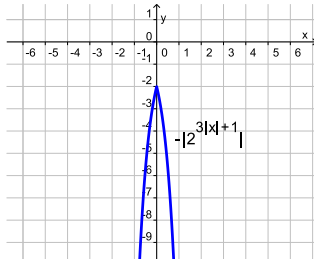
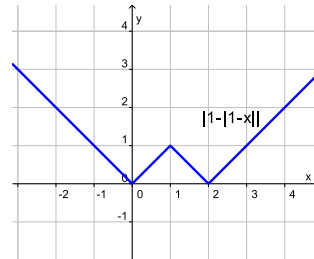
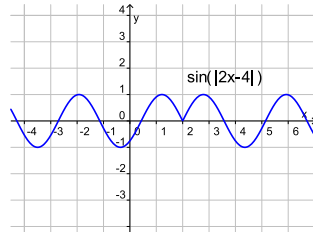
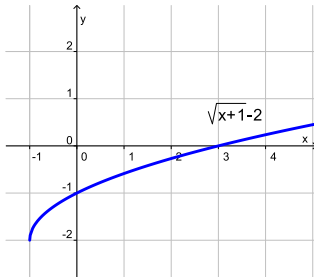
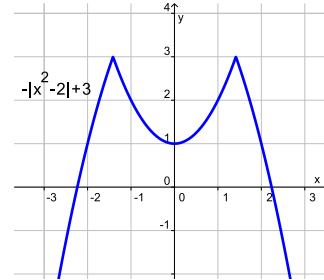
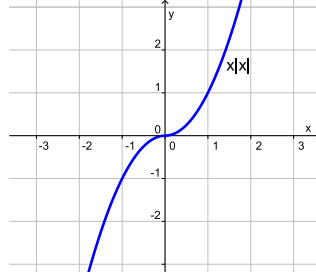
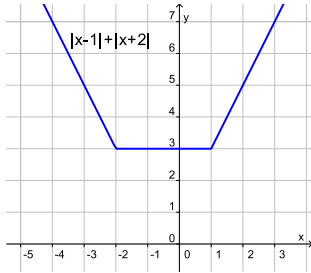
1.31 $f(3x) = 2 - 6x$

1.32 $f(t) = kt$

1.33 $f^{-1} : \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{-2\}, f(y) = \frac{2y}{1-y}$

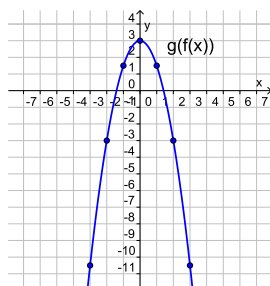
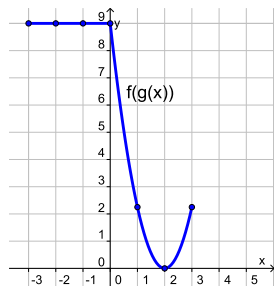
1.34 $a = 1, b = 2, c = \frac{2}{7}, d = \frac{3}{14}\pi$

1.35

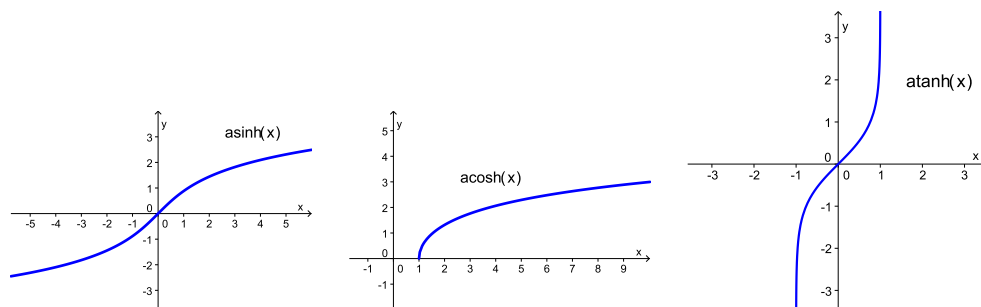


1.36

| c | $f(g(c))$ | $g(f(c))$ |
|-----|---------------|-----------------|
| -3 | 9 | $-\frac{21}{2}$ |
| -2 | 9 | -3 |
| -1 | 9 | $\frac{3}{2}$ |
| 0 | 9 | 3 |
| 1 | $\frac{9}{4}$ | $\frac{3}{2}$ |
| 2 | 0 | -3 |
| 3 | $\frac{9}{4}$ | $-\frac{21}{2}$ |



1.37 L'unica funzione biunivoca è $\sinh x$, $\tanh x$ non è suriettiva.



1.38 $xy = 3$

1.39 • $x > 0, x \neq \{1, 4\}$

• $2k\pi < x < \pi + 2k\pi, k \in \mathbb{Z}$

• $x < 0 \vee x > 1$

1.40 • $-17 \leq x < -1 \vee x > 1$

• $x < \frac{-1 - \sqrt{5}}{2} \vee -1 < x < 0 \vee \frac{-1 + \sqrt{5}}{2} < x < 1$

• $x < \frac{5}{4}$

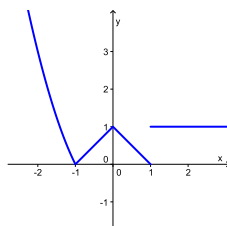
• \emptyset

• $x < \frac{\log_5 13}{3}$

• $0 + \frac{2k\pi}{3} < x < \frac{\pi}{18} + \frac{2k\pi}{3} \vee \frac{5}{18} + \frac{2k\pi}{3} < x < \frac{2}{3}\pi + \frac{2k\pi}{3}, k \in \mathbb{Z}$.

1.41

$\text{Dom}(f) = \mathbb{R}, \text{Im}(f) = [0, +\infty[$



Capitolo 2

- 2.7** a) $-3 - \frac{8}{3}i$ c) $\frac{\sqrt{37}}{3} + \sqrt{10}$ e) $\frac{\sqrt{73}}{3}$
 b) $\frac{19}{30}$ d) $\frac{\sqrt{37}}{3}(1 - 3i)$
- 2.8** a) $r = 2, \theta = \frac{\pi}{3}$ c) $r = 13\sqrt{2}, \theta = \frac{3}{4}\pi$
 b) $r = 5, \theta = \frac{5}{6}\pi$ d) $r = \frac{\sqrt{2417}}{14}, \theta = \arctan\left(\frac{49}{4}\right) - \pi$
- 2.9** a) $-527 + 336i$ b) $7 + 24i, -7 - 24i$
- 2.10** a) $-\frac{13}{29}$ c) $-\frac{2}{37}$
 b) -2
- 2.11** a) $r = 1, \theta = \frac{\pi}{2}$ c) $r = \sqrt{\frac{13}{5}}, \theta = \arctan\left(\frac{1}{8}\right) - \pi$
 b) $r = 4\sqrt{10}, \theta = \pi - \arctan\left(\frac{1}{3}\right)$
- 2.12** $z_1 = \sqrt[4]{41} \left[\cos\left(\frac{1}{2} \arctan\left(\frac{4}{5}\right)\right) - i \sin\left(\frac{1}{2} \arctan\left(\frac{4}{5}\right)\right) \right]$
 $z_2 = \sqrt[4]{41} \left[-\cos\left(\frac{1}{2} \arctan\left(\frac{4}{5}\right)\right) + i \sin\left(\frac{1}{2} \arctan\left(\frac{4}{5}\right)\right) \right]$
 $w_{1,2} = \frac{1}{4}(3 \pm i\sqrt{7})$
- 2.13** $z_1 = -2i, z_2 = 3i$
- 2.14** 2
- 2.15** a) $4 \left[\cos\left(\frac{\pi}{9} + \frac{2k}{3}\pi\right) + i \sin\left(\frac{\pi}{9} + \frac{2k}{3}\pi\right) \right], k = 0, 1, 2$
 b) $\sqrt[4]{3}, -\sqrt[4]{3}, i\sqrt[4]{3}, -i\sqrt[4]{3}$
 c) $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$
- 2.16** $12 + 8i$
- 2.17** • -3
 • 3

Capitolo 3

- 3.23** Gli anagrammi di AIUOLE sono $6!$, quelli di AIUOLA sono $\frac{6!}{2}$.
- 3.24** 210 **3.27** $\frac{8}{2907}, \frac{\binom{17}{n-4}}{\binom{21}{n}}$ **3.30** $\frac{5}{9}$
- 3.25** 5712 **3.28** $x_1 = 8, x_2 = -1$ **3.31** $D_{20,7}, 20!$
- 3.26** $\sim 0.25\%, \sim 0.025\%$ **3.29** 15 **3.32** $\frac{7!}{2}$

3.33 • 0.08123% • 0.08220% • 99.917%

3.34 101,501 **3.36** $\frac{1}{37^2}, \frac{1}{37}$

3.35 $\frac{1}{54145}$ **3.37** 240

Capitolo 4

4.35 No, $\begin{bmatrix} 0 & 0 \\ -2 & -1 \\ -5 & -1 \\ 1 & -5 \end{bmatrix}$, $\begin{bmatrix} 1 & -2 & 2 & -4 \\ 0 & 21 & 0 & 21 \end{bmatrix}$, $\begin{bmatrix} 1 & -3 & -4 & 2 \\ 0 & 14 & 7 & -14 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 \\ -4 & 42 \\ 4 & 0 \\ -8 & 42 \end{bmatrix}$, No, $\begin{bmatrix} \frac{1}{2} & -4 \\ 0 & \frac{49}{2} \end{bmatrix}$

4.36 a) $[-3, 2, -10]$ b) -5 c) 1 d) 0

4.37 $k = -4$, in generale: $a = t - s$, $b = -s$, $c = s$, $d = t$.

4.38 2160, $\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$

4.39 $\begin{bmatrix} 0 & \frac{1}{3} \\ \frac{1}{4} & -\frac{1}{12} \end{bmatrix}$, $\begin{bmatrix} 1 & 4 \\ 3 & 0 \end{bmatrix}$

4.40 (a) singolare
(b) simmetrica

(d) diagonale (triangolare superiore, inferiore e simmetrica)

(c) triangolare inferiore e singolare

4.41 $k = \frac{1 \pm \sqrt{29}}{2}$

4.42 $\det A = -5$, le altre: 5, 0, 5, 10

4.43 per ogni $k \neq 0$

4.44 $\{7, 2\}, \emptyset$

4.45 • Se $k \neq -1$ determinato; se $k = -1$ impossibile; per $k = 0$, $\left(3, \frac{5}{4}, 0\right)$

• Se $k \neq \frac{\sqrt{5} \pm 1}{4}$ determinato; se $k = \frac{\sqrt{5} \pm 1}{4}$ impossibile; per $k = 0$, $(1, 0, -1)$

• Se $k \neq \{-1, \frac{1}{2}\}$ determinato; se $k = -1$ impossibile; se $k = \frac{1}{2}$ indeterminato; per $k = 0$, $(0, 0, 0)$

• Se $k \neq \{-5, 2\}$ determinato; se $k = 5$ impossibile; se $k = 2$ indeterminato; per $k = 0$, $\left(\frac{3}{5}, -\frac{4}{5}, -\frac{4}{5}\right)$

4.46 $(2, -1, -1)$

4.47 $k = 1$

4.48 $(6, -4), (14, 2), \emptyset, (2, 2), (-2, 2)$.

4.49 Se $t = 0$, $b_{12} = 0$ e $2b_{22} = 2b_{11} + 3b_{21}$.

Se $t \neq 0$, $b_{12} = \frac{t}{2}b_{21}$ e $2b_{22} = 2b_{11} + 3b_{21}$.

4.50 $x = 1$

4.51 $x_1 = 10 - 3x_4$

4.52
$$\begin{bmatrix} 8 & 0 \\ -84 & 64 \end{bmatrix}$$

4.53 $p = 7$, No.

4.54 $t = -\frac{4}{3}, \frac{3}{2}$.

Capitolo 5

5.10 a) $+\infty$

b) $-\infty$

c) 0

d) 2

e) 0

f) 1

g) $-\frac{3}{2}$

h) $+\infty$

i) $+\infty$

l) $-\infty$

m) $+\infty$

n) 1

o) e^{-2}

p) $\frac{\sqrt{3}}{2}$

q) 0

r) Indeterminato

s) 0

5.11 a) $\forall p \in \mathbb{R}$, b) $p \leq \frac{3}{2}$

Capitolo 6

6.17 a) 1

b) 0^+

c) $-\frac{1}{2}$

d) 1

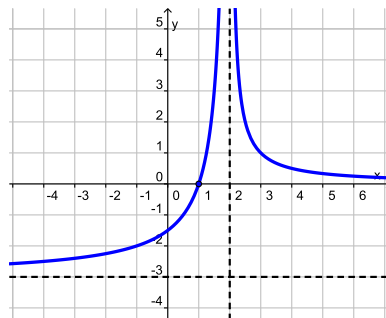
e) 0

f) $\frac{1}{\ln 2}$

g) $-\frac{1}{\sqrt{3}}$

h) 1

6.18



6.19 a) 5 b) $-\infty$ c) 0^+ d) 0^- e) $y = 0, x = 0$

6.20 $k = \frac{1}{2}, \forall k \in \mathbb{R}$

6.22 Nell'intervallo $[0, 1]$. Ne esiste un'altra nell'intervallo $[-4, -2]$.

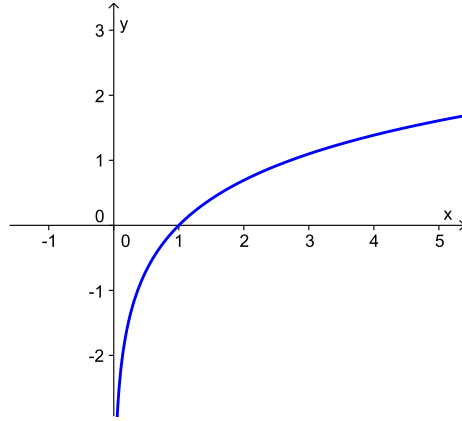
6.23 $x \simeq 0.682328$

6.24

- $k = 0$
- $a = b = 0 \vee a = -1, b = -2$
- $k = e^2$
- $k = -2$

Capitolo 7

7.32 Sì, si consideri ad esempio il seguente grafico di funzione:



- 7.33
- $-2x \cos(\cos(x^2 + 1)) \sin(x^2 + 1)$
 - $\frac{e^x(x^2 - 1) + 2x \arccos(e^x)}{(\arccos(e^x))^2}$
 - $2x^2 \sin(x^2) \left(2x \cos(x^2) \ln x + \frac{\sin(x^2)}{x} \right)$
 - $x^x \left(-\frac{\sin(\ln x)}{x} + (1 + \ln x)(\cos(\ln x)) \right)$
 - $\frac{3 + \frac{x}{1+x^2} - 3 \arctan x}{x^4}$
 - $\frac{2xe^{x^2}}{\cos^2(e^{x^2})} \sqrt[3]{x^2} + \frac{2 \tan(e^{x^2})}{3 \sqrt[3]{x}}$
 - $\frac{2f(e^x)f'(e^x)e^x \ln(f(x)) - (f(e^x))^2 \frac{f'(x)}{f(x)}}{\ln^2(f(x))}$
 - $-3 \cos(f(\sqrt{x})) \sin(f(\sqrt{x})) f'(\sqrt{x}) \frac{1}{2\sqrt{x}}$

7.34 $a = -8, b = 12, c = d = 0$.

7.35 $\text{Dom}(f) = \mathbb{R}, \text{Im}(f) = [-1, 1[$, continua, derivabile su $\mathbb{R} \setminus \{0\}$; l'equazione della retta è $y = \frac{4}{9}x - \frac{1}{9}$

7.36 $[2\sqrt{2 - \sqrt{3}}, 2\sqrt{2 + \sqrt{3}}]$, massimi assoluti in $(\pm\sqrt{2}, \frac{1}{2})$. Minimi assoluti in $(\pm 1, 0)$.

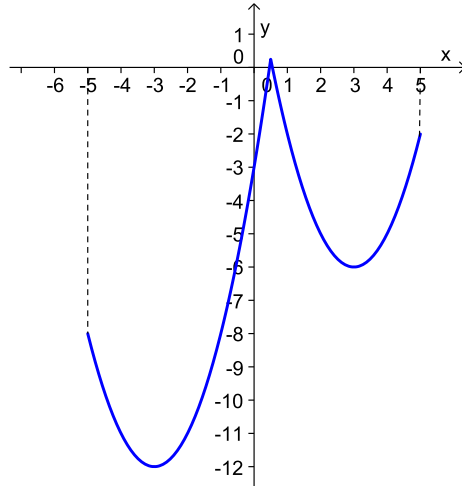
7.37 Suriettiva, non iniettiva, non periodica. Punti critici di ascissa $-\frac{\pi}{6} + k\pi$, punti di flesso di ascissa $-\frac{\pi}{3} + k\pi, k \in \mathbb{Z}$.

7.38 $b = \frac{4\sqrt{7} - 5}{6}$

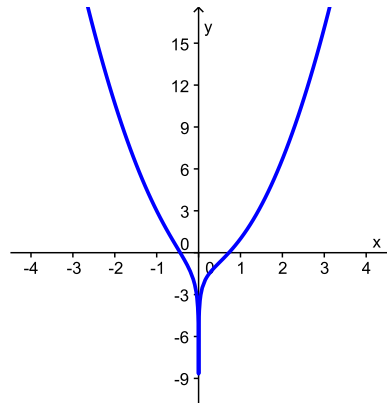
7.39 Non derivabile in $x = 0$. Punti di minimo: $(\pm 1, 0)$; punti di massimo: $(0, \sqrt{2})$.

7.40 $e + \frac{e(x-4)}{2} + \frac{e(x-4)^3}{48} + o((x-4)^3)$; non esiste il polinomio di MacLaurin.

7.41 $\left[-12, \frac{1}{4}\right]$



- 7.42** Asintoto verticale: $x = 0$; la funzione non ha massimi né minimi relativi, ha due radici, una negativa in $[-1, 0]$, l'altra positiva in $[0, 1]$. $\frac{d}{dy} f^{-1}(1) = \frac{1}{4}$



7.43 $|x - 1| + |x - 3|$

- 7.44** (a): la funzione è continua su tutto \mathbb{R} , derivabile su $\mathbb{R} \setminus \{-\frac{3}{2}, \frac{3}{2}\}$, ha un massimo relativo in $x = 0$, non ammette massimo assoluto, non ci sono asintoti; è simmetrica rispetto all'asse y .

(b): la funzione è continua su tutto \mathbb{R} , derivabile su $\mathbb{R} \setminus \{1, 2\}$, ha un minimo relativo nell'intervallo $]1, 2[$, in $x = 2$ ha un flesso verticale.

(c): la funzione è continua su $\mathbb{R} \setminus \{\frac{\pi}{2} + k\pi\}$, derivabile su $\mathbb{R} \setminus \{k\frac{\pi}{2}\}$, non ammette massimo assoluto, gli asintoti verticali sono $y = \frac{\pi}{2} + k\pi$; è simmetrica rispetto all'asse y .

- 7.45** Continua e derivabile per ogni $\alpha \in \mathbb{R}$.

Capitolo 8

- 8.24**
- | | | |
|---------------------------|---|---|
| a) $\frac{10}{9}\sqrt{5}$ | d) $2 - \frac{3}{e}$ | g) 0 |
| b) $\frac{35}{72}$ | e) $\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1$ | h) $\frac{1}{12}$ |
| c) $2\sqrt{2} - 2$ | f) $\frac{1}{2}(e^9 - e^{a^2})$ | i) $\frac{\sqrt{2}}{2} + \frac{\operatorname{arcsinh}(1)}{2}$ |

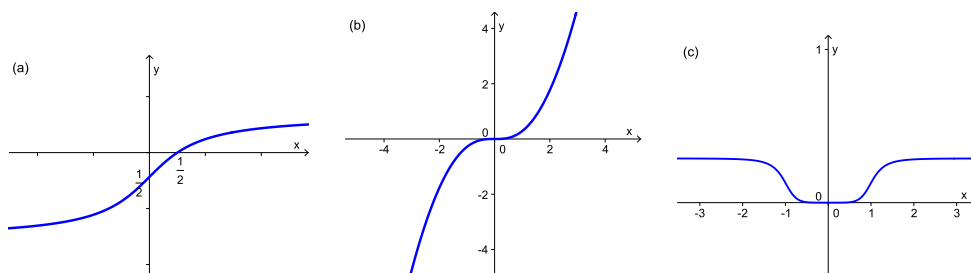
- 8.25 a) $\frac{1}{4}$ c) $\frac{1}{3}$ e) 0
 b) 1 d) $-\frac{\pi}{16}$ f) 0

8.26 $y = ex - e$

8.27 $x^2 + \frac{2}{e}x^3 + o(x^3)$

- 8.28 a) divergente d) convergente per $t < 0 \vee \frac{1}{2} < t < 2$
 b) divergente e) convergente per $p < -\frac{1}{2}$
 c) convergente f) divergente

8.29



8.30 Area = $\frac{4}{3}$

8.31 $b = 0, a = 1$

8.32 1

Capitolo 9

- 9.3 1) convergente 4) divergente 7) divergente
 2) convergente 5) convergente 8) convergente
 3) $\frac{633}{70}$ 6) divergente 9) convergente

- 9.4
- mai
 - $p < 4$
 - $x < -7 \vee x > -1, x = \frac{4 - 15\pi}{15\pi - 1}$
 - $t < -2 \vee t > -1, t = -\frac{11}{12}$
 - $x < 0$
 - $4 - \sqrt{65} < x < 4 - 3\sqrt{7} \vee 4 + 3\sqrt{7} < x < 4 + \sqrt{65}$
 - $p < 2, p \neq 1$
 - $t > \frac{11}{20}$
 - $t > 1$

- $t > \frac{3}{4}$
- $a > 0$
- $a < \frac{1}{2}$

9.5 27

Capitolo 10

10.1 50%

$$10.2 \quad 1 + \left(-\frac{2}{e^2} - \frac{1}{e}\right) x^2 + o(x^2)$$

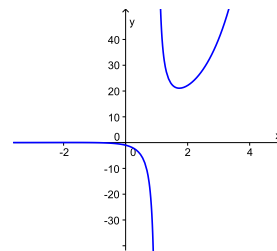
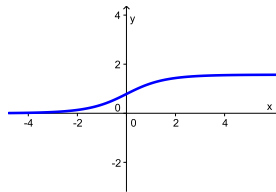
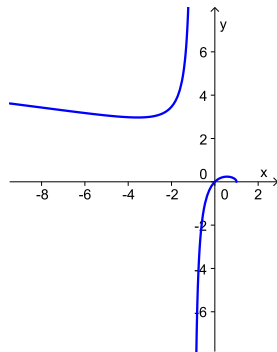
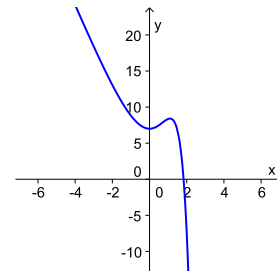
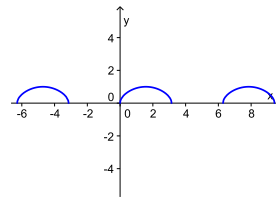
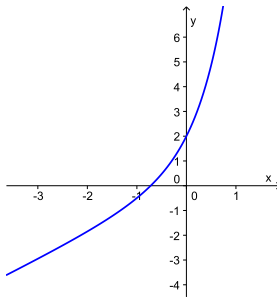
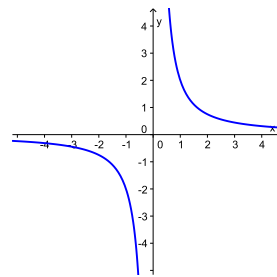
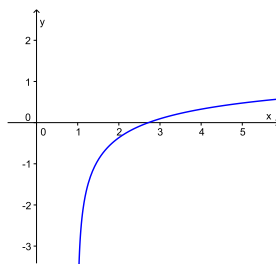
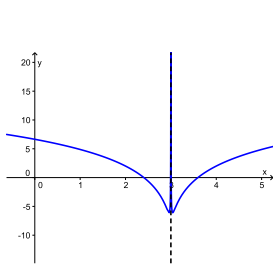
$$10.3 \quad y = -\frac{1}{2}x + 2$$

$$10.4 \quad \sup A = 1, \inf A = 0$$

$$10.5 \quad \text{Im}(f) = \left[-\frac{1}{3e} + 2, +\infty\right] \quad \text{flesso: } \left(\frac{1}{\sqrt[6]{e^5}}, -\frac{5}{6\sqrt[6]{e^5}} + 2\right)$$

$$10.6 \quad \frac{4}{21}(328 + \sqrt[4]{8})$$

10.7



10.8 $x^2 + 2x^3 + ax + b$

10.12 $3b^2 > 8ac$

10.11 42

10.13 1

10.14 1. -1

5. 0

9. $-\frac{1}{12}$

2. 2

6. 0

10. -1

3. 0

7. $\frac{1}{2}$

11. 1

4. $-\frac{\sqrt{2}}{4}$

8. $\frac{1}{2e}$

12. 2

10.15 $(8\sqrt{10} + 8) \times (10\sqrt{10} + 10)$

10.28 $a = -\frac{3}{2}$

10.16 $\frac{7}{25}$

10.29 $\frac{4}{17}$

10.17 2

10.30 (1)

10.18 0

10.31 6

10.19 $\frac{1}{\sqrt{1+x^3}}$

10.32 15

10.20 $\frac{3}{2}$

10.33 3

10.21 0

10.34 $\frac{\binom{11}{5}}{\binom{90}{5}}$

10.22 1002

10.35 $\frac{1191}{10}$

10.23 $\frac{147}{2} + \ln 2$

10.36 0

10.24 (3)

10.37 $a = 1$

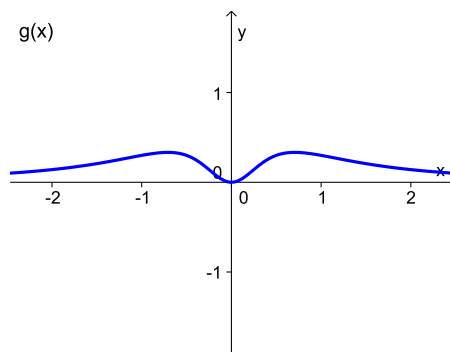
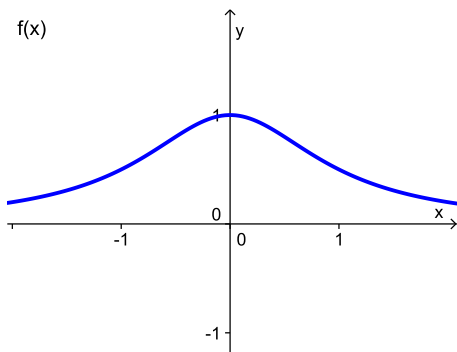
10.25 Convergente a 0

10.38 $(z, -z, z)$

10.27 $x = 0, x = \frac{15 \pm \sqrt{73}}{38}$

10.39 (4)

10.40



10.41 $-5 - 12i, 5 + 12i$

10.44 $1 + 4\pi$

10.42 $p < 0$

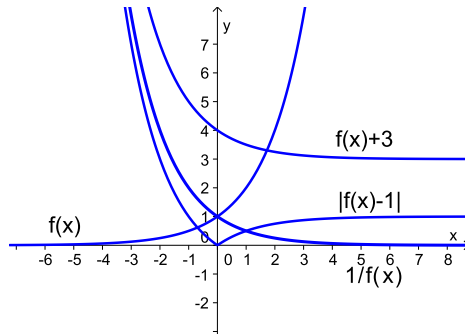
10.45 1

10.43 $-\frac{x^3}{3} + o(x^3)$

- 10.46**
- | | |
|--|-------------------|
| 1. convergente, $\frac{\sqrt{3}}{49(7-\sqrt{7})}$ | 5. convergente, 1 |
| 2. $p < 6$ | 6. convergente |
| 3. convergente | 7. convergente |
| 4. $x < -2 \vee x > 2$, $\frac{1-x^2}{x^2- x -2}$ | 8. divergente |
| | 9. divergente |

- 10.47**
- Se $k < -36$, una soluzione positiva; se $-36 < k < 0$, due negative una positiva; se $0 < k < \frac{400}{27}$, due positive una negativa; se $k > \frac{400}{27}$, una negativa.
 - Se $k \lesseqgtr \bar{k}$ una soluzione negativa; se $k \gtrreqless \bar{k}$ una negativa e due positive.

10.48



10.49 $r = \sqrt[3]{\frac{165}{\pi}}$, $h = 2\sqrt[3]{\frac{165}{\pi}}$

10.50 Nessuna

10.51 $\text{Dom}(f) = \mathbb{R}$, sempre crescente, $f(x) = 0 \Leftrightarrow x = 0$

10.52 0

- 10.53**
- Se $k \neq 2$ determinato, se $k = 2$ indeterminato, 1 parametro libero.
 - Se $k \neq -1$ determinato, se $k = -1$ impossibile.
 - Se $k \neq \{0, 2\}$ determinato, se $k = \{0, 2\}$ impossibile.
 - Se $k \neq \{-1, 3\}$ determinato, se $k = \{-1, 3\}$ impossibile.
 - Se $k \neq 5$ determinato, se $k = 5$ indeterminato, 1 parametro libero.
 - Se $k \neq -2$ determinato, se $k = -2$ indeterminato, 1 parametro libero.

10.54 Domenica

10.55 Un quadrato di lato $\sqrt{2}$

10.56 $\begin{bmatrix} 8 & 0 \\ 27 & 343 \end{bmatrix}$

10.57 $a = 2$, $b = 6\pi$, $c = 2$, $d = -\frac{\pi}{2}$

10.58 -6

10.59 $\frac{a}{b} = \frac{1 + \sqrt{5}}{2}$

10.60 $(3x + 1)^{4x} \left[4 \ln(3x + 1) + \frac{4x}{3x + 1} \right]$

10.61 k pari

10.62 $\frac{2}{\sqrt{5}}(2 - i), \frac{2}{\sqrt{5}}(i - 2)$

10.63 $x = 2$

10.65 -1.18847 o 0.066667

10.64 $k = 1$

10.66 $\left\{ \frac{1}{2}, 1 \right\}$ e $\left\{ \frac{2}{3}, 2 \right\}$